

# Relating high-energy lepton-hadron, proton-nucleus and nucleus-nucleus collisions through geometric scaling

Néstor Armesto, Carlos A. Salgado and Urs Achim Wiedemann  
*Department of Physics, CERN, Theory Division, CH-1211 Genève 23, Switzerland*  
 (Dated: February 2, 2008)

A characteristic feature of small- $x$  lepton-proton data from HERA is geometric scaling – the fact that over a wide range of  $x$  and  $Q^2$  all data can be described by a single variable  $Q^2/Q_{\text{sat}}^2(x)$ , with all  $x$ -dependence encoded in the so-called saturation momentum  $Q_{\text{sat}}(x)$ . Here, we observe that the same scaling ansatz accounts for nuclear photoabsorption cross sections and favors the nuclear dependence  $Q_{\text{sat},A}^2 \propto A^\alpha Q_{\text{sat}}^2$ ,  $\alpha \simeq 4/9$ . We then make the empirical finding that the same  $A$ -dependence accounts for the centrality evolution of the multiplicities measured in Au+Au collisions at RHIC. It also allows to parametrize the high- $p_t$  particle suppression in d+Au collisions at forward rapidities. If these geometric scaling properties have a common dynamical origin, then this  $A$ -dependence of  $Q_{\text{sat},A}^2$  should emerge as a consequence of the underlying dynamical model.

1. All data for the photoabsorption cross section  $\sigma^{\gamma^*p}(x, Q^2)$  in lepton-proton scattering with  $x \leq 0.01$  have been found [1] to lie on a single curve when plotted against the variable  $Q^2/Q_{\text{sat}}^2$ , with  $Q_{\text{sat}}^2 \sim x^{-\lambda}$  and  $\lambda \simeq 0.3$ . To further explore this empirical property of *geometric scaling*, we study here how experimental data on lepton-nucleus collisions constrain the geometric information entering  $Q_{\text{sat}}^2$ . We also ask to what extent the geometric scaling ansatz can account for characteristic features of particle production in other nuclear collision systems.

Geometric scaling is usually motivated in the QCD dipole model [2] where the total  $\gamma^*h$  cross section reads

$$\sigma_{T,L}^{\gamma^*h}(x, Q^2) = \int d^2\mathbf{r} \int_0^1 dz |\Psi_{T,L}^{\gamma^*}(Q^2, \mathbf{r}, z)|^2 \sigma_{\text{dip}}^h(\mathbf{r}, x). \quad (1)$$

Here  $\Psi_{T,L}$  are the perturbatively computed transverse and longitudinal wave functions for the splitting of  $\gamma^*$  into a  $q\bar{q}$  dipole of transverse size  $\mathbf{r}$  with light-cone fractions  $z$  and  $(1-z)$  carried by the quark and antiquark respectively. Both, for a proton [ $h = p$ ] and for a nucleus [ $h = A$ ],  $\sigma_{\text{dip}}^h(\mathbf{r}, x)$  can be written as an integral of the dipole scattering amplitude  $N_h$  over the impact parameter  $\mathbf{b}$ ,

$$\sigma_{\text{dip}}^h(\mathbf{r}, x) = 2 \int d^2\mathbf{b} N_h(\mathbf{r}, x; \mathbf{b}). \quad (2)$$

In this setting, geometric scaling corresponds to the condition  $N_h(\mathbf{r}, x; \mathbf{b}) \equiv N(rQ_{\text{sat}}(x, \mathbf{b}))$ . This can be seen by rescaling the impact parameter in (1) in terms of the radius  $R_h$  of the hadronic target,  $\bar{\mathbf{b}} = \mathbf{b}/\sqrt{\pi R_h^2}$ ,

$$\begin{aligned} \sigma_{T,L}^{\gamma^*h}(x, Q^2) &= \pi R_h^2 \int d^2\mathbf{r} \int_0^1 dz |\Psi_{T,L}^{\gamma^*}(Q^2, \mathbf{r}, z)|^2 \\ &\times 2 \int d^2\bar{\mathbf{b}} N_h(rQ_{\text{sat}}(x, \bar{\mathbf{b}})). \end{aligned} \quad (3)$$

Since  $|\Psi_{T,L}^{\gamma^*}(Q^2, \mathbf{r}, z)|^2$  is proportional to  $Q^2$  times a function of  $\mathbf{r}^2 Q^2$ , Eq. (3) depends solely on  $\tau =$

$Q^2/Q_{\text{sat}}^2(x, \bar{\mathbf{b}})$ . In the case of  $\gamma^*-A$  interactions, geometric scaling is the property that the  $A$ -dependence of the ratio  $\sigma_{T,L}^{\gamma^*h}/\pi R_h^2$  can be absorbed in the  $A$ -dependence of  $Q_{\text{sat},A}(x, \bar{\mathbf{b}})$ ,

$$\frac{\sigma^{\gamma^*A}(\tau_A)}{\pi R_A^2} = \frac{\sigma^{\gamma^*p}(\tau_p = \tau_A)}{\pi R_p^2}. \quad (4)$$

For this  $A$ -dependence, we make the ansatz that the saturation scale in the nucleus grows with the quotient of the transverse parton densities to the power  $1/\delta$ ,

$$Q_{\text{sat},A}^2 = Q_{\text{sat},p}^2 \left( \frac{A\pi R_p^2}{\pi R_A^2} \right)^{\frac{1}{\delta}} \Rightarrow \tau_A = \tau_h \left( \frac{\pi R_A^2}{A\pi R_h^2} \right)^{\frac{1}{\delta}}, \quad (5)$$

where the nuclear radius is given by the usual parametrization  $R_A = (1.12A^{1/3} - 0.86A^{-1/3})$  fm. We treat  $\delta$  and  $\pi R_p^2$  as free parameters to be fixed by data.

2. In Fig. 1 we plot the experimental  $\gamma^*p$  data [3] with  $x \leq 0.01$  as a function of  $\tau = Q^2/Q_{\text{sat}}^2$ . For  $Q_{\text{sat}}^2$ , we use in this plot the Golec-Biernat and Wüsthoff (GBW) parametrization [4] with  $R_0^2 = 1/Q_{\text{sat}}^2 = (\bar{x}/x_0)^\lambda$  in  $\text{GeV}^{-2}$ ,  $x_0 = 3.04 \cdot 10^{-4}$  and  $\lambda = 0.288$ . To safely extend to low virtuality, the  $x$ -dependence of the GBW parametrization is modified by a mass term  $\bar{x} = x \left( \frac{Q^2 + 4m_f^2}{Q^2} \right)$  with  $m_f = 0.14$  GeV. The data [3] are seen to be parametrized well by the scaling curve

$$\sigma^{\gamma^*p}(x, Q^2) \equiv \Phi(\tau) = \bar{\sigma}_0 [\gamma_E + \Gamma(0, \xi) + \ln \xi], \quad (6)$$

where  $\gamma_E$  is the Euler constant,  $\Gamma(0, \xi)$  the incomplete  $\Gamma$  function and  $\xi = a/\tau^b$ , with  $a = 1.868$  and  $b = 0.746$ . The normalization is fixed by  $\bar{\sigma}_0 = 40.56$  mb. The functional shape of (6) can be motivated by making simplifying assumptions in evaluating (1). For our purpose, however, Eq. (6) is just a convenient ansatz for the scaling function  $\Phi(\tau)$ .

To determine  $Q_{\text{sat},A}^2$ , we compare the functional shape of (6) to the available experimental data for  $\gamma^*A$  collisions with  $x \leq 0.0175$  [5, 6, 7], using  $\xi = a/\tau_A^b$ . The

parameters  $\delta$  and  $\pi R_p^2$  in (4) – (6) are fitted by  $\chi^2$  minimization adding the statistical and systematic errors in quadrature. The data sets [5], [6] and [7] have additional normalization errors of 0.4%, 0.2% and 0.15%; the quality of the fit improves by multiplying the data by the factors 1.004, 1.002 and 0.9985 respectively. We obtain  $\delta = 0.79 \pm 0.02$  and  $\pi R_p^2 = 1.55 \pm 0.02 \text{ fm}^2$  for a  $\chi^2/\text{dof} = 0.95$  – see Fig. 1 for comparison. If the normalizations are all set to 1, we obtain an almost identical fit with  $\delta = 0.80 \pm 0.02$  and  $\pi R_p^2 = 1.57 \pm 0.02 \text{ fm}^2$  for a  $\chi^2/\text{dof} = 1.02$ . If we impose  $\delta = 1$  in the fit, which corresponds to  $Q_{\text{sat,A}}^2 \propto A^{1/3}$  for large nuclei, a much worse value of  $\chi^2/\text{dof} = 2.35$  is obtained. We conclude that the small- $x$  experimental data on  $\gamma^*A$  collisions favor an increase of  $Q_{\text{sat,A}}^2$  faster than  $A^{1/3}$ . The numerical coincidence  $b \simeq \delta$  is consistent with the absence of shadowing in nuclear parton distributions at  $Q^2 \gg Q_{\text{sat,A}}^2$ .

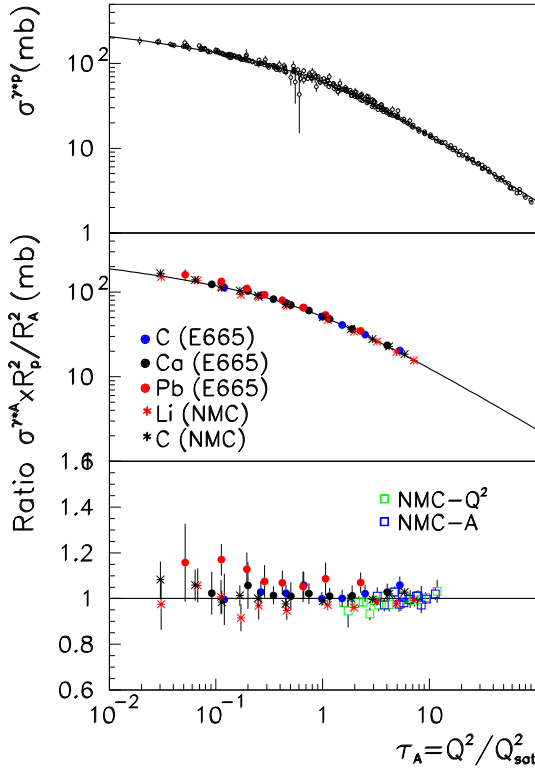


FIG. 1: Geometric scaling for  $\gamma^*p$  (upper panel, data from [3]),  $\gamma^*A$  (middle panel, data from [5, 6]) and the ratio of data for  $\gamma^*A$  divided by the scaling curve (6) (lower panel). Also shown in the lower panel are the data from [7] for ratios over  $C$ .

3. Can geometric scaling, and in particular the  $A$ -dependence and energy dependence of  $Q_{\text{sat,A}}(x)$ , account for the  $p_t$ -integrated multiplicity in symmetric nucleus-nucleus collisions at mid-rapidity? To address this question, we turn now to the heuristic ansatz

$$\left. \frac{dN^{AA}}{dy} \right|_{y \sim 0} \propto Q_{\text{sat,A}}^2 \pi R_A^2, \quad (7)$$

which arises in several models of hadroproduction [8, 9, 10, 11]. These models relate the parton distribution measured in  $\sigma^{\gamma^*A}$  to the hadroproduction measured in nucleus-nucleus collisions. For example, the factorized formula [8] calculates gluon production by convoluting  $A$ -dependent gluon distribution functions

$$\frac{dN_g^{AB}}{dy d^2 p_t d^2 b} \propto \frac{\alpha_S}{p_t^2} \int d^2 k \phi_A(y, k^2, b) \phi_B(y, (k - p_t)^2, b), \quad (8)$$

where  $\phi_h(y, k, b) = \int d^2 r \exp\{i \mathbf{r} \cdot \mathbf{k}\} N_h(\mathbf{r}, x; \mathbf{b}) / (2\pi r^2)$  [9]. For geometric scaling,  $\phi_A(y, k^2, b) \equiv \phi(k^2/Q_{\text{sat,A}}^2(y, b))$ , we find the dependence of Eq. (7),

$$\begin{aligned} \left. \frac{dN^{AA}}{dy} \right|_{y \sim 0} &\propto \int \frac{d^2 p_t}{p_t^2} dk^2 d^2 b \phi\left(\frac{k^2}{Q_{\text{sat,A}}^2}\right) \phi\left(\frac{(k - p_t)^2}{Q_{\text{sat,A}}^2}\right) \\ &= Q_{\text{sat,A}}^2 \pi R_A^2 \int \frac{d^2 s}{s^2} d^2 \tau d^2 \bar{b} \phi(\tau^2) \phi((\tau - s)^2). \end{aligned} \quad (9)$$

Also without invoking factorization in (8), any integrand with  $(k/Q_{\text{sat,A}})$ -scaling leads to Eq. (7), see [10, 11]. In all these models, a one-to-one correspondence between parton and hadron yields is assumed.

To write (7) in measurable quantities, we express the energy dependence of the saturation scale in terms of the GBW parameter  $\lambda = 0.288$ , and we translate its  $A$ -dependence to an  $N_{\text{part}}$  dependence fixed by our fit parameter  $\delta = 0.79 \pm 0.02$ ,

$$\frac{1}{N_{\text{part}}} \left. \frac{dN^{AA}}{d\eta} \right|_{\eta \sim 0} = N_0 \sqrt{s}^\lambda N_{\text{part}}^{\frac{1-\delta}{3\delta}}. \quad (10)$$

The overall normalization, independent of the energy and the centrality of the collision, is fixed to  $N_0 = 0.47$ . As seen in Fig. 2, this reproduces without further adjustment experimental data from the PHOBOS Collaboration [12] on charged multiplicities in Au+Au collisions at  $\sqrt{s} = 19.6, 130$  and  $200 \text{ GeV/A}$ . Even the  $\bar{p}p$  data ([13], as quoted in [12]) at  $\sqrt{s} = 19.6$  and  $200 \text{ GeV}$  are accounted for by Eq. (10). In the same figure, we show the result of (10) for intermediate RHIC energy ( $\sqrt{s} = 62.5$ ), for LHC energy ( $5500 \text{ GeV/A}$ ) and for smaller colliding nuclei. Eq. (10) implies that the energy and the centrality dependence of the multiplicity factorize, in agreement with the results by PHOBOS [12].

4. In the current debate of RHIC data on the suppressed high- $p_t$  hadroproduction in nuclear collisions, the relevance of nuclear shadowing has been discussed repeatedly [14, 15]. It is clear by now [16] that the  $A$ -dependence of  $p_t$ -differential hadroproduction in nucleus-nucleus collisions and in deuteron-nucleus collisions at mid-rapidity both involve additional nuclear effects which are at least as significant as nuclear shadowing. On the other hand, arguments have been put forward [14, 15] that in  $d+Au$  collisions at forward rapidity, nuclear shadowing may be the dominant effect. Motivated by the

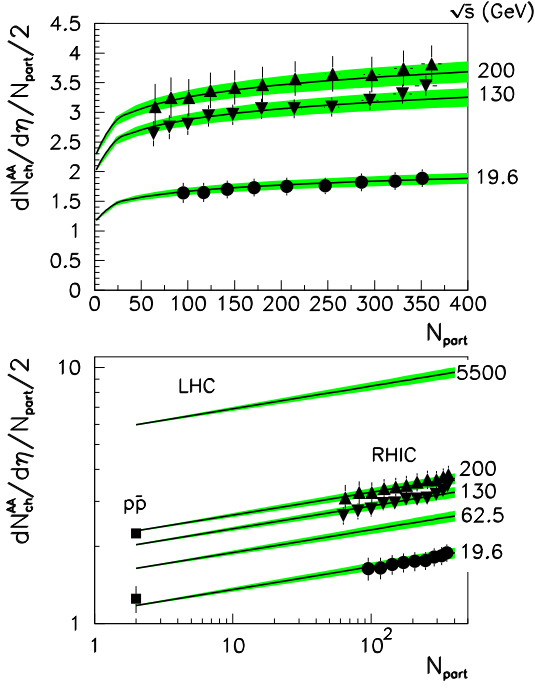


FIG. 2: Energy and centrality dependence of the multiplicity of charged particles in AuAu collisions (10) compared to PHOBOS data [12]. Also shown in the lower panel are the  $p\bar{p}$  data [13] and results for  $\sqrt{s} = 62.5$  and 5500 GeV/A.

phenomenological success of the scaling ansatz (10), we now test to what extent the centrality dependence of  $p_t$ -differential hadron spectra in d+Au emerges naturally from the geometric scaling found in  $\gamma^*A$ . We start from the model (8), using for the  $k_t$ -differential nuclear gluon distribution the scaling function  $\Phi(\tau)$  at virtuality  $\Phi(\tau) \simeq \phi(k = Q/2)$ , such that  $\tau = k^2/4\bar{Q}_{sat,A}^2$ ,  $\bar{Q}_{sat,A}^2 = N_c Q_{sat,A}^2/C_F$ . This approximation can be motivated in a momentum space representation [17] of (1). The parton distribution in the deuteron is taken to fall off sufficiently quickly,  $\sim 1/k_t^n$ ,  $n \gg 1$ , so that we can write

$$\frac{dN_{ch}^{dAu}}{N_{coll_1} d\eta d^2p_t} \approx \frac{N_{coll_2} \phi_A(p_t/Q_{sat_1})}{N_{coll_1} \phi_A(p_t/Q_{sat_2})} \approx \frac{N_{coll_2} \Phi(\tau_1)}{N_{coll_1} \Phi(\tau_2)}. \quad (11)$$

We see the use of this rough pocket formula mainly in emphasizing the plausible claim that the suppression of d+Au at forward rapidity traces directly the suppression of nuclear parton distributions at small  $x$ . For the comparison in Fig. 3 to data [18] on the normalized yields of central and semi-central over peripheral dAu collisions, we use the number of collisions  $N_{coll}$  in different centrality bins [18]  $13.6 \pm 0.3$ ,  $7.9 \pm 0.4$  and  $3.3 \pm 0.4$ . Only the two most forward rapidities  $\eta = 2.2$  and  $3.2$  are compared. We find that Eq. (11) captures main features of the recent data by BRAHMS [18] but it shows a weaker rapidity dependence. A more quantitative discussion is

certainly beyond the accuracy of (11). The only conclusion from this exercise is that the more differential analysis of (8) is not inconsistent with data in d+Au.

5. We now comment on the differences with other approaches. The geometric scaling in  $\gamma^*A$  data has been studied in [19], where a growth of  $Q_{sat,A}^2 \propto A^\alpha$ ,  $\alpha \leq 1/3$  has been found. This disagreement with our finding could have several origins. First,  $0.01 < x < 0.1$  was allowed in [19]; however, in this antishadowing region, we find no scaling in the data, as expected. Moreover, [19] does not modify the variable  $\bar{x}$  for small- $Q^2$  as done in this work and in the GBW model. Second, [19] uses  $R_A \propto A^{1/3}$  which leads to differences in particular for small  $A$  – we find a much worse fit in terms of  $\chi^2$  for such an ansatz. In [19] this disagreement is improved by introducing a free parameter  $\gamma$  in the  $A$ -dependent normalization of the nuclear  $F_2^A$  data,  $A^{-\gamma-1}$ . In our case, however, this normalization is fixed by a dimensionful quantity given by the scaling condition (4).

The multiplicities in Au+Au collisions at RHIC have been studied [20] on the basis of Eq. (8) assuming  $Q_{sat,A}^2 \propto A^{1/3}$ . These authors are lead to an expression which is Eq. (10) with  $\delta = 1$  times an additional factor  $\ln(\sqrt{s}^\lambda N_{part}^{1/3})$  argued to come from scaling violations. We note that for the accessible range of  $A$ ,  $A^{4/9} \sim A^{1/3} \ln(A^{1/3})$  – this is the reason why both approaches provide a fair description of the data at RHIC. However, the energy dependence in the logarithmic prefactor introduced in [20] implies a flatter centrality dependence with increasing energy – this difference to Eq. (10) becomes sizable at higher  $\sqrt{s}$ .

Finally, the connection between the small  $x$ - and  $A$ -dependence of parton distribution functions, and the suppression of normalized yields in d+Au collisions [18] at forward rapidity has been discussed in several recent works [14, 15]. Eq. (11) contributes to this discussion by illustrating to what extent the suppression of high- $p_t$  particles in d+Au at RHIC can be accounted for by the shadowing in  $\gamma^*A$  collisions ( $\Phi = \sigma^{\gamma^*A}$ ).

6. Here, we have discussed to what extent data for different collision systems in  $\gamma^*A$ ,  $dA$  and  $AA$  can be related through geometric scaling. Our study does not exclude the possibility that geometric scaling in  $\gamma^*p$  and  $\gamma^*A$  is a numerical coincidence without any dynamical origin. However, geometric scaling also emerges naturally in non-linear small- $x$  QCD evolution equations [21, 22], which allow to absorb the entire dependence of small- $x$  parton distributions on energy and geometry into a single dimensionfull quantity,  $Q_{sat}$ . The data discussed here are currently considered [16] to provide the main support for such non-linear saturation effects. In fact, the scaling function  $\Phi$  in (6) resembles the asymptotic solution of the Balitsky-Kovchegov (BK) equation: it behaves as  $\ln(k/Q_{sat}) [(Q_{sat}^2/k^2)^b]$  for small [large]  $k$  [14, 23]. Given that these non-linear evolution equations hold in a novel high-density regime of QCD which may become

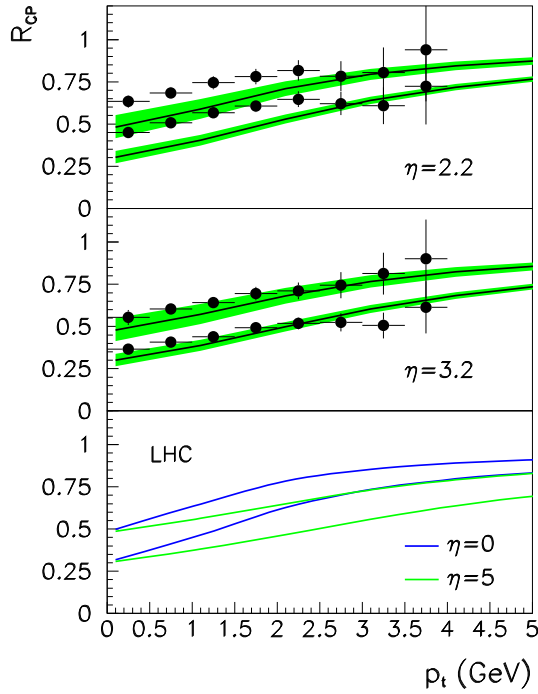


FIG. 3: Normalized ratios of central and semi-central to peripheral dAu collisions measured by BRAHMS [18] compared to results from Eq. (11). The bands represent the uncertainty in the determination of  $N_{\text{coll}}$  [18]. Results for the same centrality classes at the LHC are given in the lower panel.

experimentally accessible, it is of obvious interest to ask whether the connection between geometric scaling in the theory and in the data can be made more quantitative. On the theoretical side, this requires at least the study of the impact parameter dependence [24] of small- $x$  evolution, and the control of higher order effects. In particular, running coupling effects are known qualitatively to decrease the energy dependence [25] and the  $A$ -dependence [26] of the saturation scale in comparison to the BK equation at fixed coupling (e.g., Refs. [26] favor a weak  $A$ -dependence  $Q_{\text{sat},A}^2 \propto A^\alpha$ ,  $\alpha \ll 1/3$  but do not address the impact parameter dependence which is expected to increase the  $A$ -dependence). While an  $A$ -dependence of  $Q_{\text{sat},A}^2 \propto A^{1/3}$  is often assumed [27], much stronger ones (such as  $\alpha \simeq 2/3$  [28]) have also been proposed. The present work has analyzed to what extent data constrain these energy- and  $A$ -dependences. These constraints have to be met by non-linear small- $x$  evolution or by any other model which aims at providing the common dynamical origin for geometric scaling in different nuclear collisions.

We thank R. Baier, M. Braun, A. Capella, D. Kharzeev, A. Kovner, L. McLerran, G. Roland, K. Rummukainen and H. Weigert for discussions.

- [2] N. N. Nikolaev and B. G. Zakharov, Z. Phys. C **49**, 607 (1991); A. H. Mueller, Nucl. Phys. B **415**, 373 (1994).
- [3] J. Breitweg *et al.*, Phys. Lett. B **487**, 53 (2000); C. Adloff *et al.*, Eur. Phys. J. C **21**, 33 (2001); C. Adloff *et al.*, Nucl. Phys. B **497**, 3 (1997); M. R. Adams *et al.*, Phys. Rev. D **54**, 3006 (1996).
- [4] K. Golec-Biernat and M. Wusthoff, Phys. Rev. D **59**, 014017 (1999).
- [5] M. R. Adams *et al.*, Z. Phys. C **67**, 403 (1995).
- [6] M. Arneodo *et al.*, Nucl. Phys. B **441**, 12 (1995).
- [7] M. Arneodo *et al.*, Nucl. Phys. B **481**, 3 (1996); *ibid.* B **481**, 23 (1996).
- [8] L. V. Gribov, E. M. Levin and M. G. Ryskin, Phys. Rept. **100**, 1 (1983).
- [9] M. A. Braun, Phys. Lett. B **483**, 105 (2000); Y. V. Kovchegov and K. Tuchin, Phys. Rev. D **65**, 074026 (2002); R. Baier, A. H. Mueller and D. Schiff, hep-ph/0403201.
- [10] Y. V. Kovchegov, Nucl. Phys. A **692**, 557 (2001).
- [11] K. J. Eskola, K. Kajantie, P. V. Ruuskanen and K. Tuominen, Nucl. Phys. B **570**, 379 (2000).
- [12] B. B. Back *et al.*, Phys. Rev. C **65**, 061901 (2002); nucl-ex/0405027.
- [13] W. Thome *et al.*, Nucl. Phys. B **129**, 365 (1977); G. J. Alner *et al.*, Z. Phys. C **33**, 1 (1986).
- [14] J. L. Albacete *et al.*, Phys. Rev. Lett. **92**, 082001 (2004).
- [15] D. Kharzeev, E. Levin and L. McLerran, Phys. Lett. B **561**, 93 (2003); R. Baier, A. Kovner and U. A. Wiedemann, Phys. Rev. D **68**, 054009 (2003); D. Kharzeev, Y. V. Kovchegov and K. Tuchin, Phys. Rev. D **68**, 094013 (2003); hep-ph/0405045.
- [16] M. Gyulassy and L. McLerran, nucl-th/0405013.
- [17] M. Braun, Eur. Phys. J. C **16**, 337 (2000); N. Armesto and M. A. Braun, Eur. Phys. J. C **22**, 351 (2001).
- [18] I. Arsene *et al.*, nucl-ex/0403005.
- [19] A. Freund, K. Rummukainen, H. Weigert and A. Schafer, Phys. Rev. Lett. **90**, 222002 (2003).
- [20] D. Kharzeev and M. Nardi, Phys. Lett. B **507**, 121 (2001); D. Kharzeev and E. Levin, Phys. Lett. B **523**, 79 (2001).
- [21] I. Balitsky, Nucl. Phys. B **463**, 99 (1996); Y. V. Kovchegov, Phys. Rev. D **60**, 034008 (1999).
- [22] N. Armesto and M. A. Braun, Eur. Phys. J. C **20**, 517 (2001); M. Lublinsky, Eur. Phys. J. C **21**, 513 (2001); S. Munier and R. Peschanski, Phys. Rev. Lett. **91**, 232001 (2003); hep-ph/0401215.
- [23] E. Iancu, K. Itakura and L. McLerran, Nucl. Phys. A **708**, 327 (2002); A. H. Mueller and D. N. Triantafyllopoulos, Nucl. Phys. B **640**, 331 (2002).
- [24] K. Golec-Biernat and A. M. Stasto, Nucl. Phys. B **668**, 345 (2003); E. Gotsman *et al.*, hep-ph/0401021.
- [25] D. N. Triantafyllopoulos, Nucl. Phys. B **648**, 293 (2003); M. A. Braun, Phys. Lett. B **576**, 115 (2003).
- [26] A. H. Mueller, Nucl. Phys. A **724**, 223 (2003); K. Rummukainen and H. Weigert, Nucl. Phys. A **739**, 183 (2004).
- [27] L. D. McLerran and R. Venugopalan, Phys. Rev. D **49**, 2233 (1994); *ibid.* D **49**, 3352 (1994).
- [28] Y. V. Kovchegov, Phys. Rev. D **61**, 074018 (2000); E. Levin and K. Tuchin, Nucl. Phys. B **573**, 833 (2000).